Graphing Secant and Cosecant Functions

Remember, $\sec x = \dfrac{1}{\cos x}$ and $\csc x = \dfrac{1}{\sin x}$.

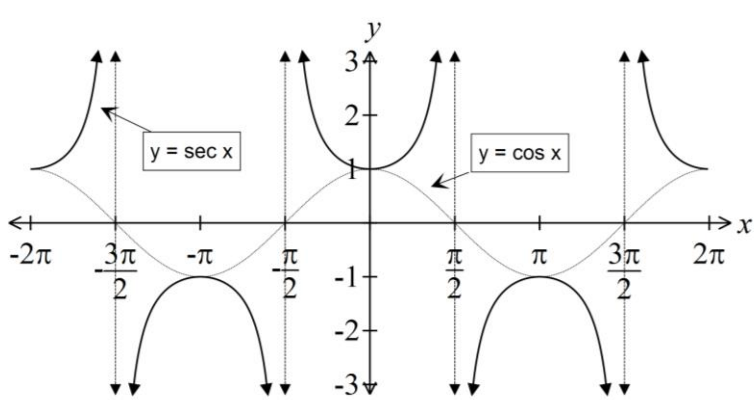
- We aren't allowed to divide by 0. This means:
  - Whenever $\cos x = 0$, $\sec x$ is undefined, and whenever $\sin x = 0$, $\csc x$ is undefined.
    - Places where $\cos x = 0$ and $\sec x$ is undefined:______________________________
    - Places where $\sin x = 0$ and $\csc x$ is undefined:______________________________
  - The graphs of $y = \sec x$ and $y = \csc x$ have vertical asymptotes at these locations.
  - **To Find the Equations of the Asymptotes:**
    - Start with any *x*-value where the function is undefined.
      Add this value to *k* times the distance between the asymptotes.
      $x = \text{asymptote} + (\text{distance between asymptotes}) \cdot k$

Graphing Secant Functions

- To graph $y = a\sec\left[b\left(x-c\right)\right]+d$:
  - Sketch the graph of $y = a\cos\left[b\left(x-c\right)\right]+d$.
  - Wherever the graph of the cosine function crosses its center point, draw a vertical asymptote.
  - The local maxima of the graph of the cosine function become local minima on the graph of the secant function with $y \to \infty$ as x approaches the asymptotes on either side. The local minima of the graph of the cosine function become local maxima on the graph of the secant function with $y \to -\infty$ as x approaches the asymptotes on either side.

Key points on the graph of $y = \sec x$:

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = \sec x$	1	undef.	-1	undef.	1

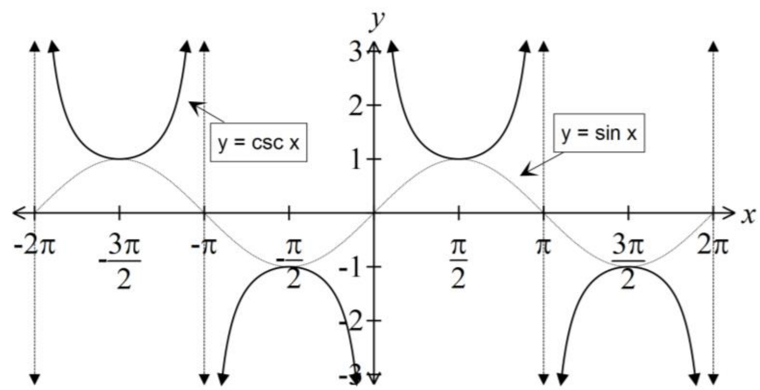


Graphing Cosecant Functions

- To graph $y = a\csc\left[b\left(x-c\right)\right]+d$:
  - Sketch the graph of $y = a\sin\left[b\left(x-c\right)\right]+d$.
  - Wherever the graph of the sine function crosses its center point, draw a vertical asymptote.
  - The local maxima of the graph of the sine function become local minima on the graph of the cosecant function with $y \to \infty$ as x approaches the asymptotes on either side. The local minima of the graph of the sine function become local maxima on the graph of the cosecant function with $y \to -\infty$ as x approaches the asymptotes on either side.

Key points on the graph of $y = \csc x$:

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = \csc x$	undef.	1	undef.	-1	undef.



Theorem 3.5. Properties of the Secant and Cosecant Functions:

- The function $F\left(x\right)=\sec\left(x\right)$
  - has domain $\left\{x : x \neq \dfrac{\pi}{2}+\pi k,\ k \text{ is any integer}\right\}$
  - has range $\left\{y : \left|y\right| \geq 1\right\} = \left(-\infty,-1\right]\cup\left[1,\infty\right)$
  - is continuous and smooth on its domain
  - is even
  - has period 2π
- The function $G\left(x\right)=\csc\left(x\right)$
  - has domain $\left\{x : x \neq \pi k,\ k \text{ is any integer}\right\}$
  - has range $\left\{y : \left|y\right| \geq 1\right\} = \left(-\infty,-1\right]\cup\left[1,\infty\right)$
  - is continuous and smooth on its domain
  - is odd
  - has period 2π

Examples: Graph the following functions. Find the period, asymptotes, and range of each.

a) $y = 3\sec(2x)$

b) $y = \csc\left(x - \frac{\pi}{4}\right) + 2$

c) $y = \sec\left(\frac{1}{2}x + \frac{\pi}{6}\right)$

d) $y = 2\csc\left(\frac{\pi}{4}x + \frac{3\pi}{4}\right)$